Diffeomorphisms preserving Morse-Bott foliations

Olexandra Khohliyk
(Department of Geometry, Topology and Dynamic Systems, Taras Shevchenko National University of Kyiv, Hlushkova Avenue, 4e, Kyiv, 03127 Ukraine)
E-mail: khokhliyk@gmail.com

Sergiy Maksymenko
(Topology Laboratory, Institute of Mathematics of NAS of Ukraine, Tereshchenkivska str., 3, Kyiv, 01004 Ukraine)
E-mail: maks@imath.kiev.ua

Let M be a smooth compact manifold and $\mathcal{F}$ be a codimension one foliation on M having singular leaves of Morse-Bott type. This means that there the set Σ of singular leaves of $\mathcal{F}$ is a disjoint union of compact submanifolds Let also $\mathcal{D}(\mathcal{F})$ be the group of diffeomorphisms of M leaving each leaf invariant, and $\mathcal{D}(\mathcal{F},\Sigma)$ be the subgroup of $\mathcal{D}(\mathcal{F})$ consisting of diffeomorphisms fixed on Σ.

Theorem 1. [1] *The "restriction to Σ map"*

$$\rho\colon \mathcal{D}(\mathcal{F}) \to \mathcal{D}(\Sigma), \qquad \rho(h) = h|_{\Sigma_f},$$

is a locally trivial fibration with fibre $\mathcal{D}(\mathcal{F},\Sigma)$.

This result can be regarded as a "foliated" variant of the well know results by Cerf and Palais on local triviality of restrictions. In particular, the map ρ has a path-lifting property, and so it contains a "foliated" variant of isotopy extension theorem:

Corollary 2. [1] *Let $H : \Sigma \times [0,1] \to \Sigma$ be a C^∞ isotopy with $H_0 = \mathrm{id}_\Sigma$. Then it extends to an isotopy $H : M\times \to M$ such that $H_t \in \mathcal{D}(\mathcal{F},\Sigma)$ for all $t \in [0,1]$.*

References

[1] Olexandra Khohliyk, Sergiy Maksymenko, *Diffeomorphisms preserving Morse-Bott functions*, arXiv:1808.03582